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# Response of roll wave to suspended load and hydraulics of overland flow on steep slope

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ABSTRACT

A roll wave is frequently observed in overland flow and it can accelerate the soil erosion on slopes. However, the feedback effect of eroded sediment on roll wave has not been studied. The aim of this study was to investigate the response of roll wave to sediment concentration and overland flow hydraulics on steep slope. The experiment was carried out in a hydraulic flume. The unit flow rate varied from 1.0 to $3.0 \times 10^{-3}$ $m^3$ $s^{-1}$, and sediment concentration from 0 to 400 kg $m^{-3}$. The sediment transport was dominated by suspension. Slope gradient was 9°. As the sediment concentration increased, the critical slope length for roll wave formation increased, implying that the suspended sediment in flow could inhibit the formation of a roll wave. The roll wave in overland flow is a short water long wave. The roll wave length increased with the increasing sediment concentration, while the wave frequency and velocity decreased. The decreased wave velocity meant a decrease in flow erosion potential caused by a roll wave. Roll wave frequency and velocity significantly increased with Reynolds number, Froude number and mean flow velocity, and decreased with the hydraulic resistance, while there were no notable relationships between roll wave length and overland flow hydraulics. Both roll wave frequency and velocity had the strongest dependency on Froude number and could be estimated by the linear equations between them. When the sediment concentration was larger than 300 kg $m^{-3}$, all the roll waves in overland flow disappeared due to the high sediment supply. The results indicated that the suspended sediment can ease the acceleration influence of a roll wave on soil erosion and should be considered in the soil erosion models.



## 1. Introduction

Free surface instabilities of flows are often observed in inclined open channels and a succession of perturbation occurs in the flow. This kind of undulating flow movement is called roll waves. A roll wave is undesired by the civil engineers, since it can periodically increase the flow depth and make the water overflow channel banks or runoff conduits which were designed to contain even more flow (Brock, 1969; Di Cristo et al., 2009). A roll wave can also cause flow intermittency at the channel outlet and produce pronounced local maxima of sediment concentration at their fronts (Liu et al., 2005). If the roll wave occurs in debris flow, the damage can be devastating since the large boulders and rocks traveling with the flow are very dangerous to the people, animals and crops in the path of the wave. It is thus worthwhile to investigate the characteristics of roll wave as well as its development process to mitigate the damage of roll wave as far as possible.

Most studies demonstrated that the formation of roll wave was closely related to flow resistance. Cornish (1934) speculated that the flow resistance played a major role in forming a roll wave, and if there were no resistance, no roll wave would happen. However, Rouse (1938) found that the roll wave would also not occur if the flow resistance were sufficiently large. Dressler (1949), Longo (2011), Smith et al. (2011) and Wang et al. (2014) emphasized that in order to obtain the roll waves, the flow resistance must be less than a certain critical value. Thomas (1937) also derived a necessary condition for roll wave formation based on the flow resistance. It can be seen that both too less or too much resistance are not conducive to the formation of a roll wave, and it only possibly occurs under a certain range of flow resistance.

Critical Froude number is widely used as the criteria separating the existence of the roll wave or not (Arai et al., 2013; Jeffreys, 1925; Smith et al., 2011; Thual, 2013). Several researchers, among them Ishihara et al. (1954), Benjamin (1957), Yih (1963, 1977) and Ferreira et al. (2015) indicated that for laminar flow down an inclined plane, the roll waves tended to form when the Froude number was greater than 0.58. Julien and Hartley (1986) found that roll wave was observed in laminar, subcritical flow at a Froude number as low as 0.74. For the turbulent flow in a rectangular channel with constant friction factor, Stoker (1957), Liggett (1975) and Armanini and Recchia (2006) found a critical Froude number of 2.0. Some other researches, like Koloseus

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and Davidian (1966) and Berlamont and Vanderstappen (1981), highlighted that the critical Froude number had strong dependency on velocity profile, Reynolds number and friction law.

Generally, the development of roll wave can be divided into three phases (Brock, 1969; Di Cristo et al., 2010; Zhang, 2011): (1) the initial development phase, in which both the roll wave period and length are relative small, the wave length varied 2–10 cm, and the waves do not overtake and combine with other waves; (2) the transitional development phase, in which roll waves begin to overtake each other, increasing their period; and (3) the mature development phase, in which the wave shape is quite obvious, and the wave period increases with the distance due to the significant roll wave coalescence. Zanuttigh and Lamberti (2002) studied the evolution of roll wave by numerical simulation, and showed that the natural roll wave cannot reach the final regime shape, but the wave height and period continuously increased with the channel due to the coalescence.

The stability of roll wave was affected by many factors, especially the flow uniformity and underlying bed. Bohorquez (2010) has shown that a small non-uniformity of the flow depth could make the flow more stable and prevent the formation of roll wave. Balmforth and Mandre (2004) explored the effect of bed topography on roll wave and found that the low-amplitude bottom topography tended to destabilize the turbulent flow and lowered the critical Froude number required for instability, while at large amplitude, the trend reversed and the onset of roll wave occurred at higher Froude number; the latter was proven by Balmforth and Vakil (2012) who reported that the stabilizing effect is much more pronounced when large bed forms are accounted for over an erodible bed. Colombini and Stocchino (2005) presented a related study of the competition between roll wave and other erosional instabilities using linear theory. The bed forms influenced the roll wave stability mainly through the hydraulic jump that often arise in flow downstream of the steepest part of the bed forms (Colombini and Stocchino, 2008; Parker and Izumi, 2000). The interaction between bed forms and roll wave implied that the sediment transport may have a significant feedback influence on roll wave, because the roll wave could affect the transport of bed and suspend sediment and localize soil erosion near the wetting front as the flow evolves downhill (Bohorquez and Fernandez-Feria, 2008), which further caused the emergence of different forms on the erodible bed.

Roll wave is more prone to form in the overland flow due to the very shallow flow depth and steep slope. The splash of raindrops can also undermine the stabilities of overland flows and promote the roll wave formation (Pan and Shangguan, 2009). The occurrence of the roll wave in overland flow has significant effects on soil erosion development, since it affects the hydraulics and hydrodynamic force distribution of the flow. Using one-dimensional St. Venant equations, Liu et al. (2005) investigated the dynamics of periodic roll wave in overland flow and indicated that the existence of roll wave could increase the flow shear stress and augmented the potential of soil erosion. Prasad et al. (2005) reported that roll wave contained a significant portion of the total kinetic energy of flow and acted as primary energy source in transporting eroded sediment in shallow flow. Zhang (2011) explored the critical conditions for flow instability in laminar and turbulent overland flow, and concluded that the critical Froude number varied between 0.5–0.7 for laminar flow and 1.59–2.22 for turbulent flow.

As mentioned above, there had existed some research in the literatures that investigated the roll wave characteristics in overland flow and the possible effect of roll wave on soil erosion and sediment transport. However, the feedback effect of the eroded sediment on roll wave has not been studied, especially at high sediment concentrations. The objectives of this study were to evaluate the potential effects of sediment concentration on roll wave characteristics, as well as the possible relationships between roll wave length, frequency/period, velocity and hydraulics of overland flow on steep slope under a wide range of sediment concentrations and hydraulic conditions.

## 2. Materials and methods

### 2.1. Experimental conditions and treatments

The hydraulic flume used in this study was 8 m long, 0.50 m wide and 0.25 m deep, with a smooth fixed bed made up of plexi-glass. The slope of the flume could be adjusted manually. Soil was collected from Yangling District, Shaanxi Province, China. The soil was air-dried, gently crushed, and then passed through a 1-mm sieve to remove gravel and residues. The particle size distribution of the test soil was shown in Table 1, with the median diameter $d_{50}$ of 0.012 mm. A 1-m$^3$ water tank, installing an electric stirring device, was used to mix the water and soil (Fig. 1(a)). Then the sediment-laden runoff was pumped into a head tank at the upper end of the flume and flowed naturally over the flume.

To simulate the influence of sediment concentration on roll wave of overland flow, twelve sediment concentrations were selected according to the slope erosion and sediment transport characteristics in the Loess Plateau of China. The sediment concentrations were 0, 30, 60, 90, 120, 150, 180, 200, 250, 300, 350 and 400 kg m$^{-3}$, respectively. Water samples were collected at the outlet of the flume to determine the actual sediment concentration. Flow discharges were 0.5, 0.75, 1, 1.25, and $1.5 \times 10^{-3}$ m$^3$ s$^{-1}$, respectively. They were controlled by a series of valves and measured directly by a calibrated flow meter on the inflow pipe. The flume was adjusted at 9°, which was a common gradient in China.

Experimental observation showed that the dominant mode of sediment transport was suspended load along the whole flume. It is evident as a result of the very low settling velocity of the particles due to the small size and the high flow velocity. Bohorquez and Fernandez-Feria (2008) also proved that the sediment transport was dominated by suspension for the particles ≤ 1.0 mm. Transient or permanent deposition of sediment occurred at the water-bed interface for some tests with the combinations of high sediment concentration and low flow discharge, but the deposition yield was very few and the particles were basically uniformly spread flat out on the whole bed (the largest average deposition thickness was only 0.12 mm). No obvious bed forms existed. For this reason we have neglected the possible bed slope change produced by the deposition as well as its influence on roll wave.

### 2.2. Experimental measurements

Prior to each test, the sediment concentration, flow rate and flume slope were adjusted to the designed values. After flow became stable, the flow depth and roll wave characteristics were measured. The side-view schematic diagram of flow in the flume and the roll wave and particle motion in flow at low and high sediment concentrations were shown in Fig. 1(b)–(d), respectively. The flow depth was measured using a digital level probe (SX40-A, Chongqing Hydrological Equipment Factory) at the section of 0.5 m above the lower end of the flume. The resolution of the digital level probe was 0.01 mm and the accuracy was 0.04 mm. For each test, nine flow depths were measured across the section and the average of the nine depths was considered as the mean flow depth of the test.

Because of the limitations of the measurement instruments, only the critical slope length for roll wave formation, roll wave length and period/frequency were measured. The critical slope length was visually observed and the value was read according to the scale marked on the flume. The roll wave length was also visually observed and measured using a steel square at the section of 7–8 m from the upper to the lower end of the flume. Six roll wave lengths were measured and the average of the six lengths was the mean roll wave length of the test. The roll wave period/frequency was measured using a digital stopwatch. The travel time of ten roll waves over every cross section (1 m interval from upslope to downslope) of the flume was recorded with five replicates. The one roll wave period was obtained by dividing the travel time

**Table 1**
Mechanical composition of the tested soil in the experiment.

| Soil type | Particle size distribution % (mm) | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1–0.5 | 0.5–0.25 | 0.25–0.05 | 0.05–0.01 | 0.01–0.005 | 0.005–0.001 | <0.001 |
| Lou soil | 0.07 | 0.65 | 5.86 | 49.04 | 12.18 | 13.72 | 18.48 |

by 10 and the average of the five values was the mean roll wave period of the cross section. The roll wave velocity was calculated from roll wave length and frequency.

During the experiment, runoff temperature was measured using a thermometer, to calculate the kinematic viscosity of the flow. The temperature varied between 20.5–24.5 °C. According to the preliminary experiment results, the duration of tests ranged from 10 to 20 min for different flow discharges, which could ensure the full development of flow and saturation of non-linear regime. Because the conditions were all relatively stable during each test, which could produce a very stable result, no repetition of the experiments was carried out. A series of 60 trials (5 flow rates × 12 sediment concentrations) were tested.

### 2.3. Data analysis

The roll wave velocity was calculated using:

$$V_w = L/T \tag{1}$$

where $V_w$ is the roll wave velocity (m s$^{-1}$), $L$ is the roll wave length (m) and $T$ is the roll wave period (s).

The mean flow velocity was calculated from mean flow depth using:

$$V = q/h \tag{2}$$

where $V$ is the mean velocity of flow (m s$^{-1}$); $h$ is the measured mean flow depth (m); $q$ is the unit flow discharge (m$^2$ s$^{-1}$).

Reynolds number ($Re$) was calculated using:

$$Re = \frac{Vh}{\nu_m} = \frac{q}{\nu_m} \tag{3}$$

where $\nu_m$ is the kinematic viscosity of silt-laden water (m$^2$ s$^{-1}$), which was calculated using (Sha, 1965):

$$\nu_m = \nu / \left(1 - \frac{S_v}{2\sqrt{d_{50}}}\right). \tag{4}$$

where $\nu$ is the kinematic viscosity of clear water (m$^2$ s$^{-1}$), which was calculated as $\nu = 0.00000178/(1 + 0.0337\,t + 0.000221\,t^2)$; $t$ is the flow temperature (°C); $S_v$ is the volumetric sediment concentration (%); $d_{50}$ is the sediment median diameter (mm).

Froude number ($Fr$) was the ratio of the velocity of the flow to its wave celerity $C$ (m s$^{-1}$), which can be expressed using (Li and Yang, 2004):

$$Fr = V/C = V/\sqrt{gh\frac{(1+\Delta h/h)^2}{(1+\Delta h/2h)}}. \tag{5}$$

where $g$ is the gravitational acceleration (m s$^{-2}$), $\Delta h$ is the wave height (m). Because of the measurement difficulty of the wave height, its effect on $Fr$ was not considered. Eq. (5) can be simplified as:

$$Fr = V/\sqrt{gh}. \tag{6}$$

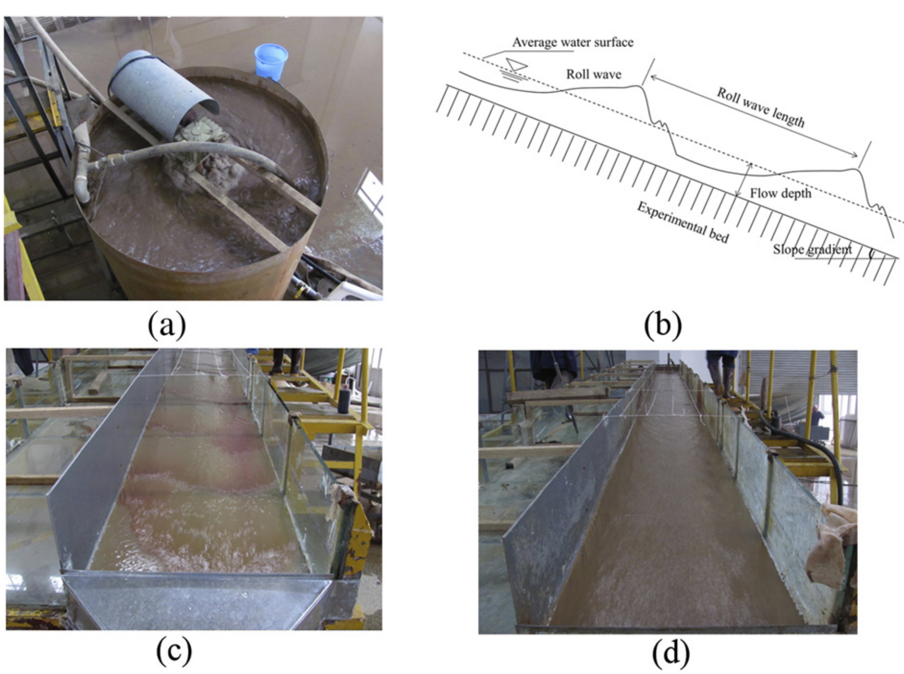


**Fig. 1.** Some pictures of the experiment and side-view schematic diagram of flow in the flume. (a) The tank supplying sediment-laden water; (b) side-view schematic diagram of flow in the flume; (c) the flow at low sediment concentration; (d) the flow at high sediment concentration.

Darcy–Weisbach (*f*) friction coefficient was calculated using:

$$f = \frac{8ghJ}{V^2} \quad (7)$$

where *J* is the flume slope (m m$^{-1}$).

## 3. Results and discussions

### 3.1. Critical slope length of roll wave formation

The development of roll wave required a minimum slope length, as pointed out by Julien and Hartley (1986) for laminar flow and by Montuori (1965) for turbulent flow. This length was called the critical slope length for the formation of the roll wave. The critical slope length varied between 1.9–5.5 m under the experimental conditions and it was affected by sediment concentration and flow rate.

As the suspended sediment concentration increased, the critical slope length increased (Fig. 2). The mean value of the critical slope length increased from 3.1 m to 4.4 m when the sediment concentration increased from 30 kg m$^{-3}$ to 150 kg m$^{-3}$, indicating that the sediment concentration could increase the distance of roll wave formation. This result implied that the suspended sediment in overland had the inhibition effect on roll wave formation. The reason may be attributed to the increasing hydraulic resistance of overland flow. According to the formation mechanism of the roll wave, when the friction of overland flow is not large enough to weaken the undulation of the uneven flow surface, the roll wave occurred. In the present study, the hydraulic resistance of overland flow increased with the sediment concentration, and the increasing hydraulic resistance could weaken the flow surface undulation, further inhibit the roll wave formation. Rouse (1938) also reported that sufficient resistance would prevent the formation of roll wave.

There were different relationships between the critical slope length and the unit flow rates for different sediment concentrations. When the sediment concentration was ≤120 kg m$^{-3}$, there was a positive relationship between the critical slope length and flow rate, and when the sediment concentration was >120 kg m$^{-3}$, the flow rate had no significant effect on the critical slope length. This result indicated that the flow rate had a greater effect on the critical slope length for low sediment concentration than that for high sediment concentration. The reason may be that the effect of suspended load on the critical slope length increased and the flow rate effect weakened under the high sediment concentration conditions.

According to the results of the open channel flow (Zhang, 2011), the critical slope length was related to the flow depth and hydraulic resistance. The best fitting equation between the critical slope length and flow depth and friction coefficient for the sediment-laden overland flow was:

$$\frac{L_c}{1000h} = 1.77f^{0.36} \quad (8)$$

where $L_c$ is the critical slope length (m). For the open channel flow, the power coefficient was about −1.0, while the value for the sediment-laden overland flow was 0.36, indicating that the effect of hydraulic resistance on the critical slope length of overland flow greatly differed from that on the open channel flow.

### 3.2. Roll wave length

The roll wave length was in the range of 0.34–0.41 m for sediment-free flow and 0.39–0.67 m for sediment-laden flow. The overland flow depth varied from 2.68 to 5.32 mm. The ratio of flow depth to roll wave length ranged from 0.006 to 0.011, which was much less than 1/20, indicating that the roll wave of overland flow was short water long wave and the effect of the roll wave could spread to the whole depth.

The roll wave length increased with the sediment concentration increasing (Fig. 3). For example, the wave length increased from 0.38 m to 0.51 m when the sediment concentration increased from 0 to 150 kg m$^{-3}$ under the unit flow rate of 1.0 × 10$^{-3}$ m$^2$ s$^{-1}$. The positive relationship between roll wave length and sediment concentration implied that long roll wave was easily formed in overland flow as the suspended load increased. Although the roll wave height was difficult to be measured during the experiment, it can be clearly observed that the wave height became small with the increasing sediment concentration, causing a decrease in the ratio of wave height to wave length. The ratio is an index that can reflect the shape of roll wave. It is possible to conclude that the roll wave in sediment-laden flow had a gentler profile than that in sediment-free flow. The flow depth also increased as the

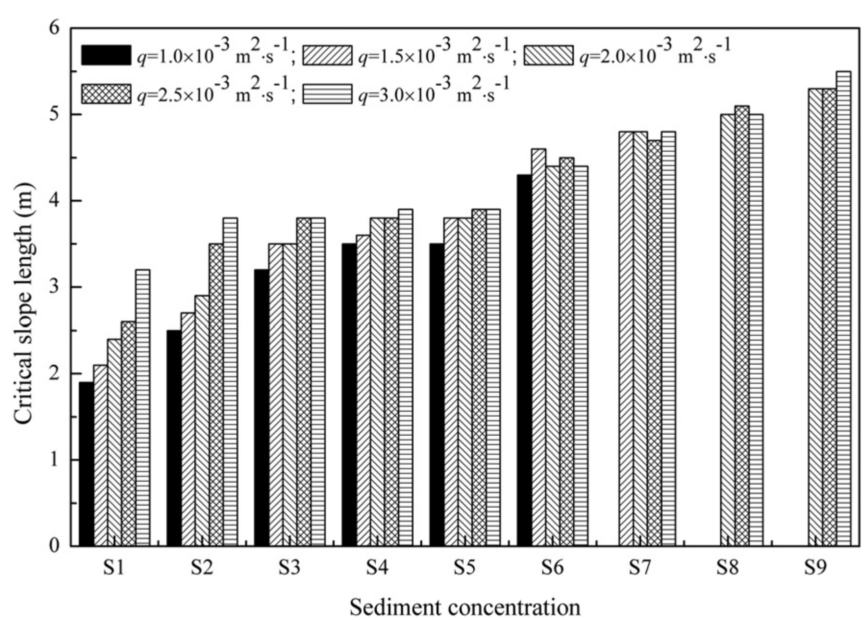


**Fig. 2.** Effects of sediment concentrations on critical slope length for roll wave formation under different flow rates. S1, S2, S3, …, S9 represent 0, 30, 60, 90, 120, 150, 180, 200, and 250 kg m$^{-3}$, respectively.

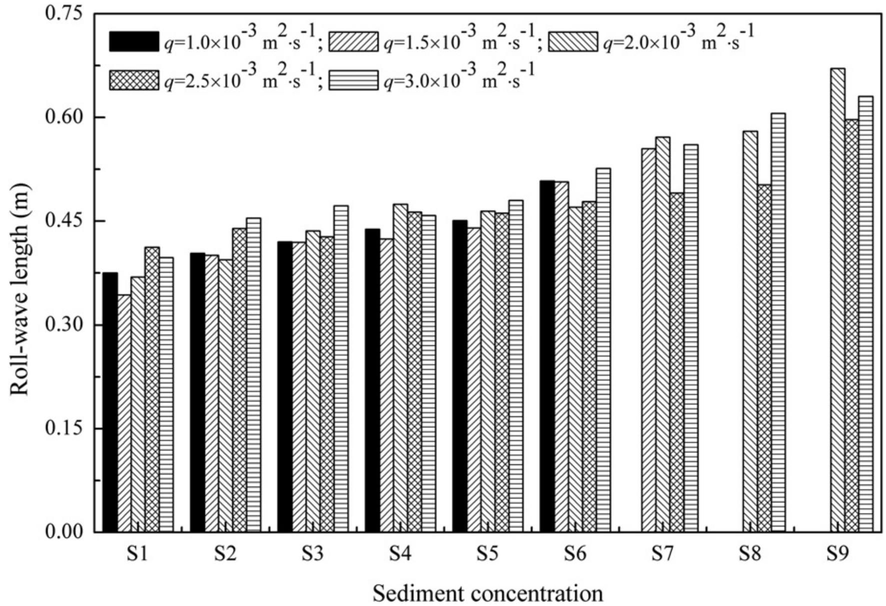


**Fig. 3.** Effects of sediment concentrations on roll wave length under different flow rates. S1, S2, S3, …, S9 represent 0, 30, 60, 90, 120, 150, 180, 200, and 250 kg $m^{-3}$, respectively.

sediment concentration increased, so the ratio of roll wave height to flow depth decreased. The ratio can reflect the development of roll wave. This result indicated that the presence of suspended sediment in flow could inhibit the roll wave development.

When the sediment concentration reached a certain value, the roll wave became less evident or gradually disappeared. The critical sediment concentration values differed with the different unit flow discharges. The value ranges were about 150–180, 180–200, 250–300, 250–300, and 250–300 kg $m^{-3}$, respectively, for the unit flow rate 1.0, 1.5, 2.0, 2.5 and $3.0 \times 10^{-3}$ $m^2$ $s^{-1}$. Clearly, the critical sediment concentration that the roll wave disappeared increased as the flow rate increased, and when the sediment concentration was larger than 300 kg $m^{-3}$, nearly

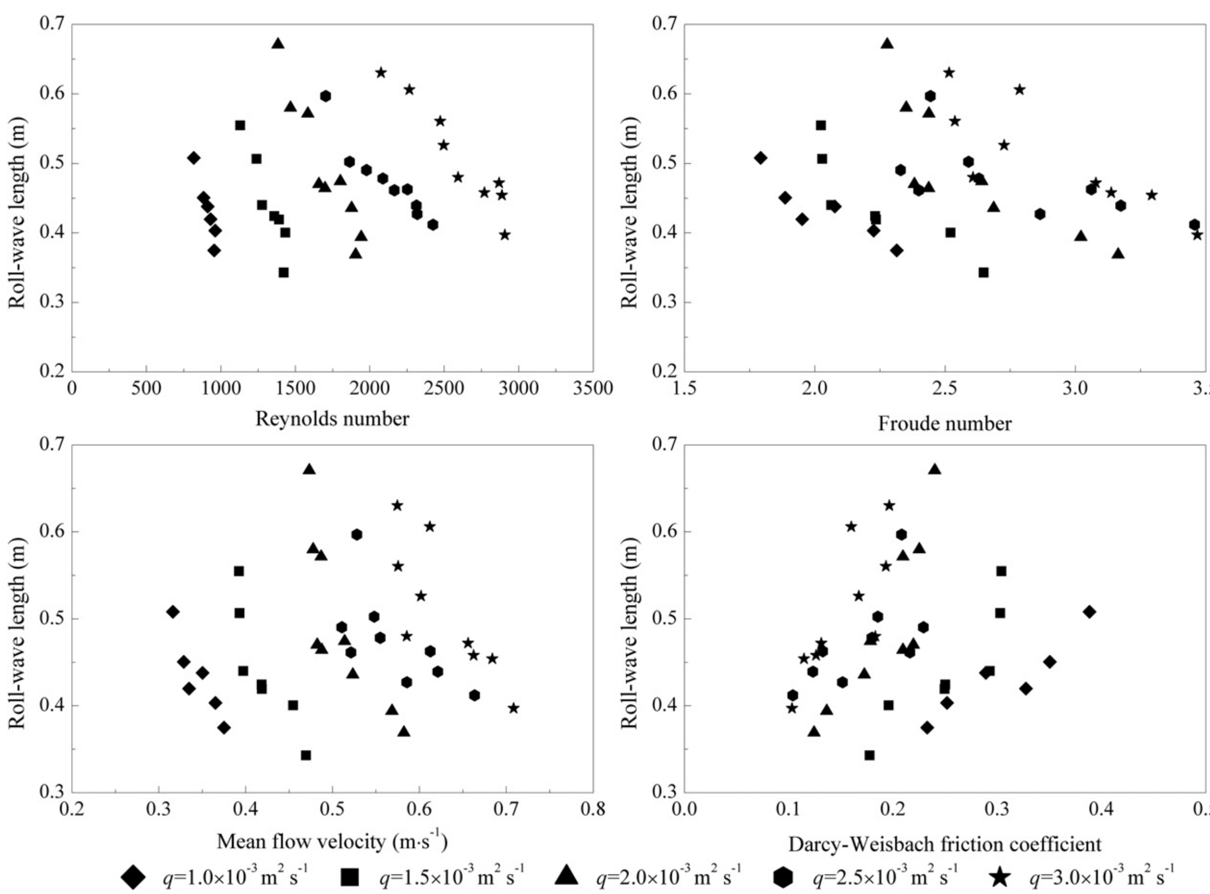


**Fig. 4.** Relationships between roll wave length and Reynolds number, Froude number, mean flow velocity and Darcy–Weisbach friction coefficient under different flow rates.

all the roll wave disappeared no matter the flow rate was. Both the increasing critical slope length of roll wave formation and high sediment supply with the increasing sediment concentration may cause the disappearance of roll waves. However, in this study, the high sediment supply should be the dominant reason: first, the flume length was enough to meet the increasing critical slope length of roll wave formation in high sediment supply conditions. Although the critical slope length of roll wave formation increased with the sediment concentration, the maximum amplification was not more than 0.8 m and the average value was 0.4 m for each unit of sediment concentration increase (30 kg $m^{-3}$ at low sediment concentration and 50 kg $m^{-3}$ at high concentration) for all tests (Fig. 2). And as the suspended load increased, the increasing rate of critical slope length decreased. The largest critical slope length before the roll wave disappearance varied 4.3–5.5 m for the five flow rates, and the higher the flow rate was, the larger the critical slope length was. For the largest flow rate $3.0 \times 10^{-3}$ $m^2$ $s^{-1}$, the maximum and average amplifications of the critical length were 0.5 m and 0.3 m, respectively. It can be inferred that the critical slope length should be about 6.0 m or less when reaching the critical sediment concentration that the roll wave disappeared, which was much smaller than the flume length of 8.0 m. For the lower flow rates, the values were smaller. The calculated results of the critical slope length using Eq. (8) at the critical sediment concentrations, in the range of 4.5–6.0 m, also confirmed the above viewpoint; second, as mentioned above, the formation of roll wave was related to flow resistance and too less or too much resistance are not conducive to its formation. As the sediment concentration increased, the hydraulic resistance of overland flow increased. When the sediment supply reached the critical sediment concentration, the flow resistance was large enough to prevent the formation of roll wave; third, the high sediment supply may change the overland flow properties. The flow became non-Newtonian fluid from Newtonian and the sediment and water separation no longer occurred due to the flocculation of cohesive particles (very small inflow sediment particle size) when the suspended load was very high. At this critical sediment concentration, the flow had the characteristics of one-phase flow and water and sediment particles began to move as a whole, thereby causing the disappearance of roll waves.

There was a parabolic variation of roll wave length with the increasing flow rate under the similar sediment concentrations (Fig. 3). The roll wave length decreased initially and increased later with the flow rate. The critical flow rate corresponding to the minimum value of the roll wave length increased with the increasing sediment concentration.

The Reynolds number and Froude number varied between 818–2906 and 1.79–3.47, respectively. The mean flow velocity and Darcy–Weisbach friction coefficient varied between 0.32–0.71 m $s^{-1}$ and 0.10–0.39, respectively. The overland flow was the turbulent, supercritical flow. In general, the roll wave length decreased as Reynolds number, Froude number and mean flow velocity increased, and increased as the hydraulic resistance increased (Fig. 4). The negative relation between roll wave length and Froude number was also found by Julien and Hartley (1986). When the flow rate was fixed, nearly all the hydraulic parameters had a significant influence on roll wave length according to the one-way ANOVA, and Reynolds number was the most relevant parameter compared with the others. However, for all tests, the effect of hydraulics of overland flow on roll wave length was not significant.

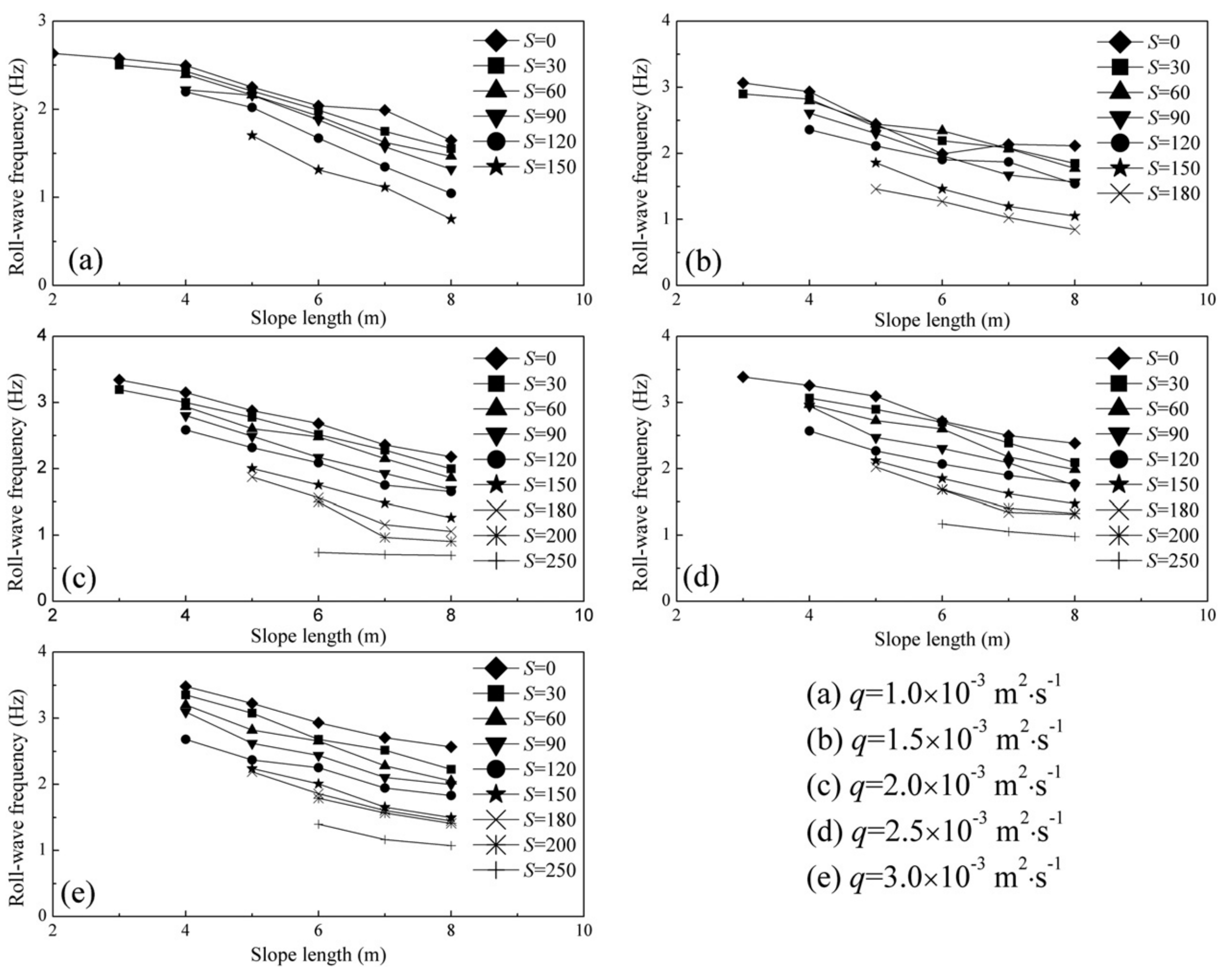


**Fig. 5.** Variation of roll wave frequency along slope length under different sediment concentrations and flow rates.

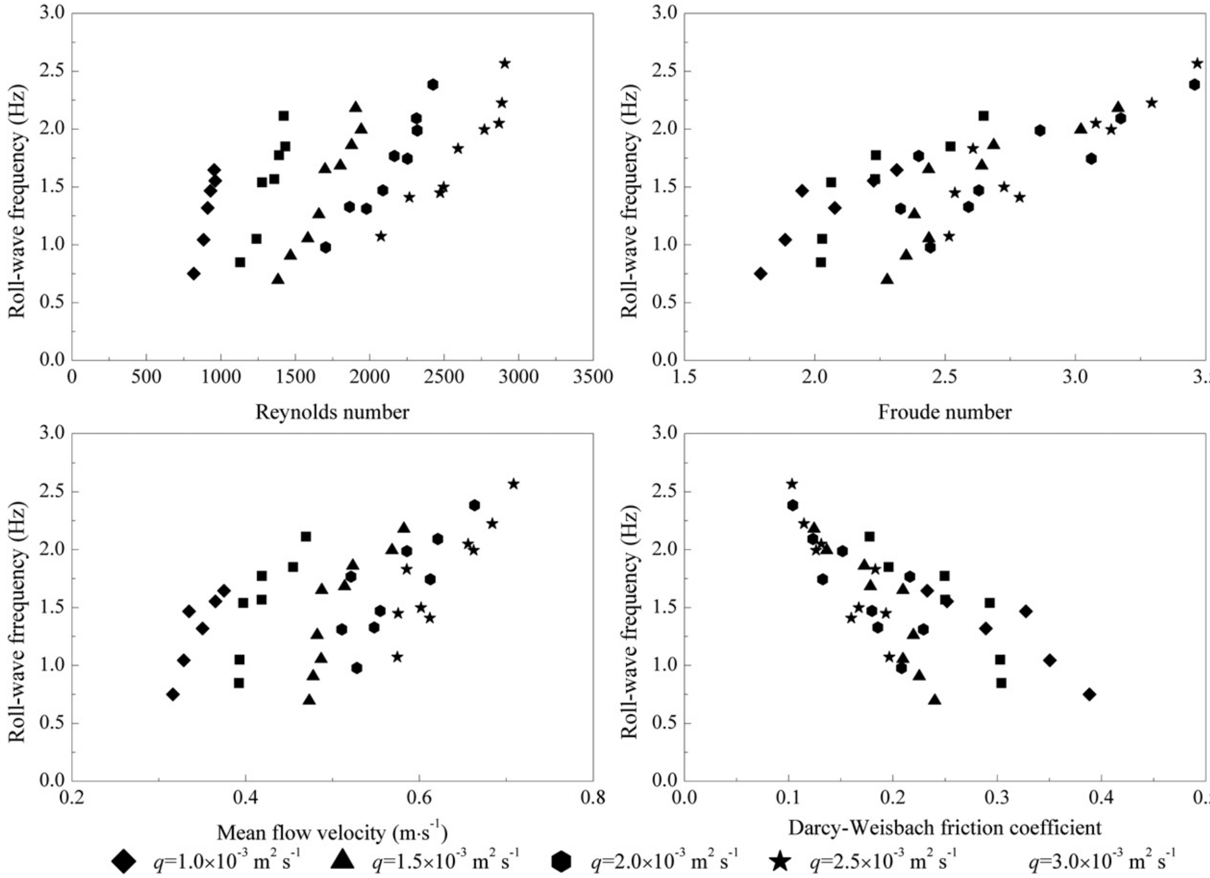


**Fig. 6.** Relationships between roll wave frequency and Reynolds number, Froude number, mean flow velocity and Darcy–Weisbach friction coefficient under different flow rates.

### 3.3. Roll wave frequency/period

The variation in roll wave frequency can reflect the coalescence of the roll wave. The roll wave frequency of overland flow continuously decreased along the slope length under different suspended sediment concentrations and flow discharges (Fig. 5). This meant that the period of roll wave increased as the slope length increased. Experimental observations also showed that the roll wave cannot remain periodically stable, and it often occurred such that the back wave caught up with the front wave and then the two waves merged into one wave. The result was consistent with the conclusion in the open channel flow (Brock, 1970; Zanuttigh and Lamberti, 2002). The variation of roll wave frequency along the slope length could lead to the variation in the hydrodynamic force distribution in overland flow, which affected the soil erosion potential on slope. The decreasing slope of roll wave frequency along the slope length was larger for the low sediment concentration than that for high sediment concentration, indicating that the roll wave had a less drastic coalescence under the high sediment concentration conditions.

As the sediment concentration increased, the roll wave frequency decreased and the wave period increased under the fixed flow rate, while the roll wave frequency increased as the flow rate increased under the similar sediment concentration conditions, corresponding to the decreasing in the roll wave period (Fig. 5). Also, the attenuation degree of roll wave frequency along the slope increased with the increasing flow rate. The average attenuation coefficient of roll wave frequency were 0.189, 0.210, 0.240, 0.244 and 0.282, respectively, for the unit flow rate of 1.0, 1.5, 2, 2.5, and $3.0 \times 10^{-3}$ $m^2$ $s^{-1}$ at the sediment concentration of 30 kg $m^{-3}$. It showed that the flow rate could strengthen the roll wave coalescence along the slope length.

The relationships between roll wave frequency at 7–8 m along the slope length and the hydraulics of overland flow were showed in Fig. 6. The roll wave frequency increased with the Reynolds number, Froude number and mean flow velocity increasing, and decreased with the friction coefficient increasing. Under each flow rate condition, the effects of four hydraulic parameters on roll wave frequency were notable. The wave frequency had the strongest dependence on Reynolds number compared with the other parameters, which was the same as the roll wave length. For all tests, the hydraulics of overland flow also had significant influences on roll wave frequency. The best possible relationships between the roll wave frequency and the four hydraulic parameters were all linear according to the regression analysis.

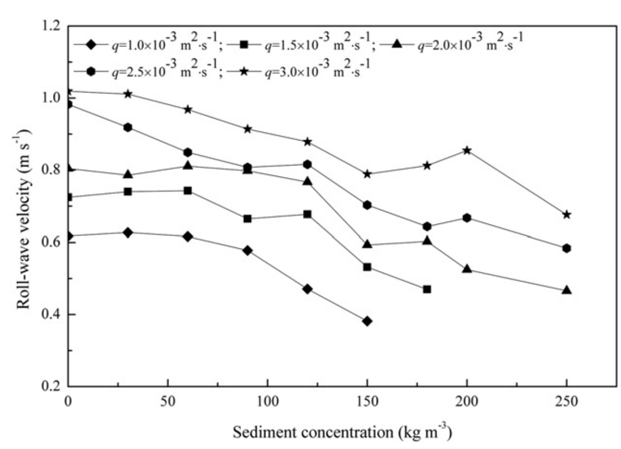


**Fig. 7.** Effects of sediment concentrations on roll wave velocity under different flow rates.

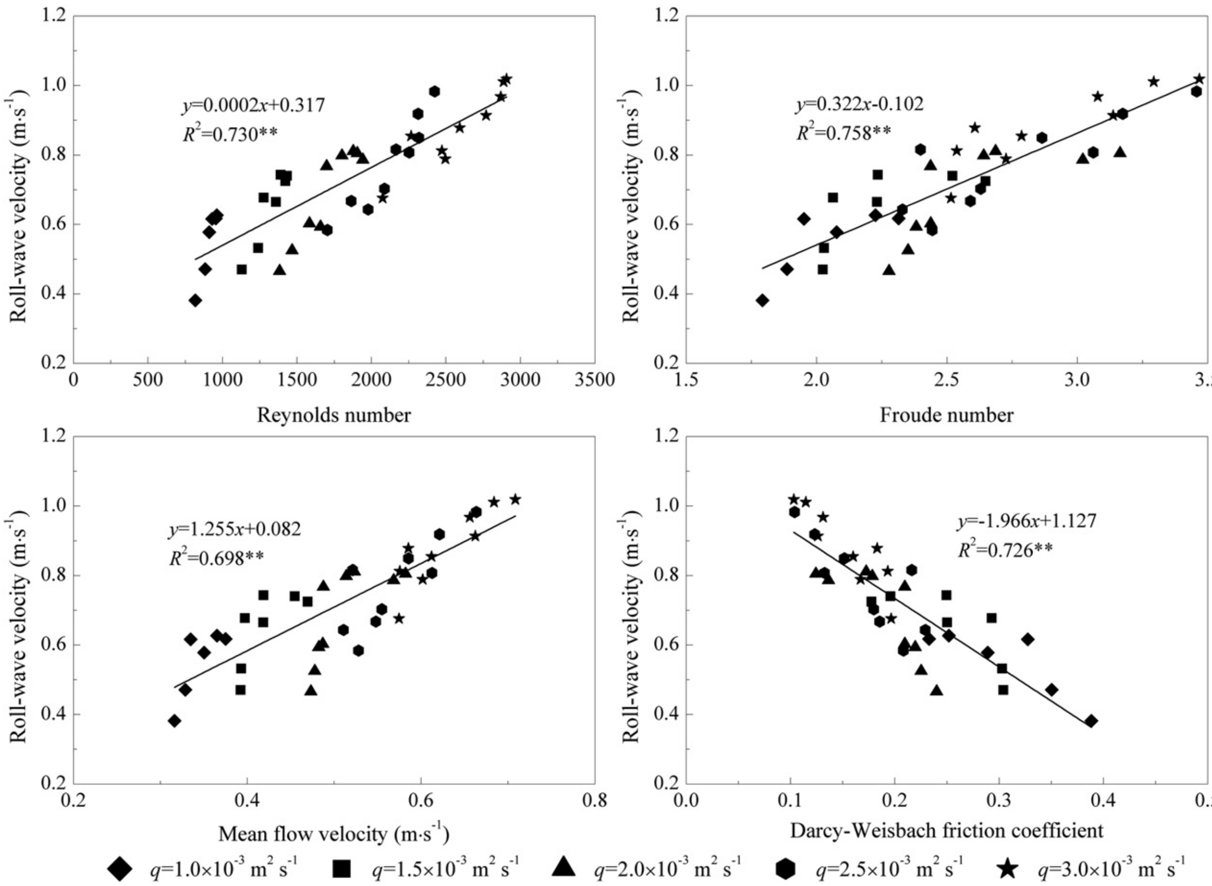


**Fig. 8.** Relationships between roll wave velocity and Reynolds number, Froude number, mean flow velocity and Darcy–Weisbach friction coefficient under different flow rates. The solid lines were the fitted lines between wave velocity and the four parameters for all tests.

Froude number was the most important factor affecting the roll wave frequency, and this may be one of the reasons why the Froude number was often selected as the criteria to determine the formation of the roll wave in flow. The frequency of roll wave in sediment-laden overland flow could be estimated by *Fr*.

### 3.4. Roll wave velocity

The roll wave velocity of overland flow decreased as the sediment concentration increased and increased as the unit flow rate increased (Fig. 7). This result was consistent with the effect of sediment concentration and flow rate on mean flow velocity of overland flow. Similar results were also found by Li and Ning (1994). The negative relationship between roll wave velocity and sediment concentration meant that the suspended sediment could decrease the potential of flow erosion, that is, the probability of soil erosion on the slope caused by the roll wave could decreased when the sediment existed in the overland flow.

Roll wave velocity increased as Reynolds number, Froude number and mean flow velocity of overland flow increased, and decreased as the hydraulic resistance increased (Fig. 8). The positive relationship between wave velocity and Froude number also found in the over-concentrated flow with intense bed load (Armanini and Recchia, 2006). Reynolds number still had the closest relationship with the roll wave velocity under the same unit flow rate.

For all tests, the hydraulics of overland flow also had significant influence on the roll wave velocity. The relationships between roll wave velocity and the four hydraulic parameters could all be described by the linear equations, and these equations can be used for the prediction of the roll wave velocity with the similar experimental conditions. Among the four parameters, Froude number played the most important role in affecting the roll wave velocity of overland flow, with the highest correlation coefficient of the fitted regression lines. The roll wave velocity is about 1.25 times of the mean flow velocity of overland flow, indicating that the unit stream power of overland flow could be increased by 25% when the roll wave occurred. This result accorded with Liu et al. (2005) who reported that the maximum shear stress can be increased by 10–30% by the roll wave. This finding was helpful in understanding soil erosion on a hillslope. It showed that when a roll wave occurred in overland flow, the overland flow exhibited different hydraulic characteristics, and soil erosion could occur more easily. This also meant the occurrence of the roll wave could accelerate the soil erosion process on slopes.

**Table 2**
Critical flow conditions that roll wave disappeared.

| $q$ ($10^{-3} \times m^2 \cdot s^{-1}$) | $S$ ($kg \cdot m^{-3}$) | $h$ (mm) | $V$ ($m \cdot s^{-1}$) | $Re$ | $Fr$ | $f$ |
|---|---|---|---|---|---|---|
| 1.0 | 150–180 | 3.17 | 0.32 | 818 | 1.79 | 0.39 |
| 1.5 | 180–200 | 3.83 | 0.39 | 1130 | 2.02 | 0.30 |
| 2.0 | 250–300 | 4.40 | 0.47 | 1383 | 2.28 | 0.24 |
| 2.5 | 250–300 | 4.76 | 0.53 | 1704 | 2.44 | 0.21 |
| 3.0 | 250–300 | 5.32 | 0.57 | 2074 | 2.52 | 0.20 |

### 3.5. Critical conditions that the roll wave disappeared

As analyzed above, the roll wave disappeared when the suspended sediment concentration in overland flow reached a certain range for the fixed flow discharge. Table 2 listed the critical flow conditions that the roll wave of sediment-laden overland flow disappeared for each flow rate. The critical values of the flow depth, mean flow velocity, Reynolds number, Froude number and friction coefficient corresponded to the hydraulic parameter of sediment-laden flow at the low limit of the critical sediment concentration range. It could be seen that as the unit flow discharge increased, the critical sediment concentration increased, and all the values were less than 300 kg $m^{-3}$. The critical flow depth, mean flow velocity, Reynolds number and Froude number also increased with the increasing flow rate, while the critical hydraulic resistance decreased. The critical Froude number varied from 1.79 to 2.52 under different flow rates conditions, and the average value was 2.21, larger than the critical Froude number of 2.0 in turbulent flow in rectangular channel (Armanini and Recchia, 2006), indicating that the suspended sediment in overland flow could increase the critical Froude number required for instability. The ratio of the critical Froude number to the corresponding value in sediment-free flow varied from 0.70 to 0.77 with the average of 0.74, which may be helpful in determining the critical sediment concentration that the roll wave disappeared under different flow discharges. Table 2 can be used as a reference to judge whether the roll wave disappeared or not in the sediment-laden flow under the similar experimental conditions.

## 4. Conclusions

This study was conducted to investigate the effect of suspended sediment concentration on the roll wave characteristics and the possible relationships between roll wave length, frequency and velocity and hydraulics of overland flow on steep slope. Results showed that the roll wave in overland flow was short water long wave and could affect the whole range of the flow depth. The critical slope length for roll wave formation and roll wave length increased as the sediment concentration increased, while the roll wave frequency and velocity decreased. This implied that the suspended sediment in overland flow can inhibit the formation of roll wave and decreased the probability of soil erosion on the slope caused by the roll wave.

In general, for all tests, roll wave frequency and velocity significantly increased as Reynolds number, Froude number and mean flow velocity increased, and decreased as the friction coefficient increased, while there were no notable relationships between roll wave length and hydraulics of overland flow. Among the four parameters, both roll wave frequency and velocity were the most closely related to Froude number, and they could be estimated by the linear equations between them. However, for the tests at the same flow rate, all the roll wave length, frequency and velocity were significantly affected by the overland flow hydraulics, and all of them had the strongest dependency on Reynolds number.

Roll wave disappeared when the sediment concentration in overland flow reached a certain value. The critical sediment concentration increased as the flow rate increased, and all values were less than 300 kg $m^{-3}$. The critical flow depth, mean flow velocity, Reynolds number and Froude number that the roll wave disappeared also increased with the increasing flow rate, while the critical hydraulic resistance decreased. The feedback effect of sediment concentration and overland flow hydraulics on roll wave characteristics was significant and this could further affect the development of soil erosion. The results should be considered in soil erosion models to improve the prediction accuracy, especially on steep arable where sediment concentrations are commonly great.

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